THERMAL REGIMES FOR SUPER-HIGH-FREQUENCY HEATING OF DIELECTRICS

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The features of thermal processes in a dielectric under the action of a super-high-frequency electromagnetic field are studied. It is shown that the nonlinear heat evolution due to the absorption of electromagnetic energy can result at a critical temperature in the development of a nonsteady thermal regime in the sample. For bodies of different geometry, the relationship is found between the critical temperature and the threshold value of the power of super-high-frequency generation and thermophysical properties of the dielectric and conditions of heat exchange with the surrounding medium.

Introduction. The propagation of a super-high-frequency (SHF) electromagnetic wave through a dielectric is related to the absorption of energy and heating of the material. The heating is followed by an increase in dielectric losses, which, in turn, results in a larger heat evolution. Therefore, the SHF heating becomes nonlinear [1]. Intense heat evolution takes place in the continually-contracting central part of the sample. The nonsteady heat regime that develops under these conditions is similar to the LS thermal regime with peaking [1].

The transition to the thermal regime with peaking should take place when the steady-state temperature distribution in the sample becomes impossible, similarly to the combustion of gas in the theory of thermal explosion [3] or the case of the thermal breakdown of dielectrics [4].

The steady-state heat conduction equation for an infinite plane plate with heat sources q, which emerge under the action of the electromagnetic field and are distributed continuously when the heat conduction of the medium is constant, is of the form

$$k\frac{d^2T}{dx^2} = -q \tag{1}$$

with the boundary conditions

$$k\left.\frac{dT}{dx}\right|_{x=\pm r} = \mp\alpha(T - T_\infty). \tag{2}$$

The solution of Eq. (1) that satisfies boundary conditions (2) describes a steady-state temperature distribution in the medium. However, if the thermoelectrophysical properties of the material are such that starting from a certain temperature $T_{cr}$ the steady thermal distribution becomes impossible, then this temperature should be considered as a temperature of transition to the thermal regime with peaking. Its value can be determined from an analysis of properties of Eq. (1).

If the power of heat evolution depends only on the temperature $q = q(T)$, then the general integral of Eq. (1) is taken by double quadrature and with account of the boundary conditions is of the form

$$r = \int_{T_s}^{T_m} \frac{dT}{\sqrt{2\int_{T}^{T_m} \frac{q(T)}{k}\,dT}} = \psi(T_m,\ T_s). \tag{3}$$

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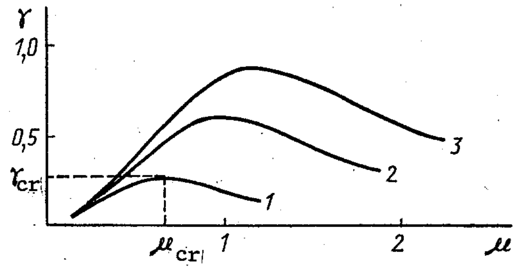


Fig. 1. Behavior of the function $\gamma = f(\mu)$ for an infinite plane plate ($b = 0$, $\Theta_\infty = 0$): 1) Bi = 1; 2) 5; 3) ∞.

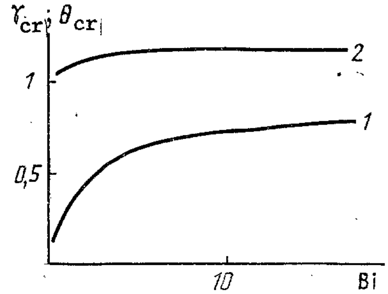


Fig. 2. The dependence, for a plane plate, of the values $\gamma_{cr}$ and $\Theta_{cr}$ on the conditions of heat exchange ($b = 0$, $\Theta_\infty = 0$): 1) $\gamma_{cr}$; 2) $\Theta_{cr}$.

D. A. Frank-Kamenetskii has shown that if $\psi(T_m, T_s)$ is a monotonic function with respect to $T_m$ then the steady-state regime is always possible. However, if the function $q(T)$ is such that when $T_m$ varies, $\psi$ passes through the extremum, then this extremum should give the critical values $r_{cr}$ and $T_{cr}$. For $r > r_{cr}$ and $T > T_{cr}$ the steady-state temperature distribution is impossible. For values of r and T less than critical, each r should correspond to the two values of $T_m$, i.e., two different steady-state temperature distributions. The stable distribution is the one with the lower value of the maximum temperature.

Critical temperature for SHF heating. The density of heat sources for normal incidence of the electromagnetic wave can be calculated, as was shown in [5], from the equation

$$q = -dS/dx. \tag{4}$$

Here $S = S_0 \exp(-2x/h_E)$ is the density of the energy flow;

$$h_E = \frac{\lambda_0}{\pi\sqrt{2\varepsilon_1\left(\sqrt{1+(\varepsilon_2/\varepsilon_1)^2}-1\right)}} \tag{5}$$

is the penetration depth of the electromagnetic wave.

In the SHF range for a number of dielectric materials (quartz and aluminosilicate ceramics) $\varepsilon_1$ changes weakly during the heating, and the temperature dependence of $\varepsilon_2$ can be approximated by the exponential function [6]

$$\varepsilon_2 = a\,[b + \exp(\beta(T - T_0))]. \tag{6}$$

Here $T_0$, a, b, and β are the approximation constants.

For most dielectrics, when the temperature dependence $\varepsilon_2$ is approximated, the constant $b > 1$, and the value of the constant component of the coefficient of dielectric losses

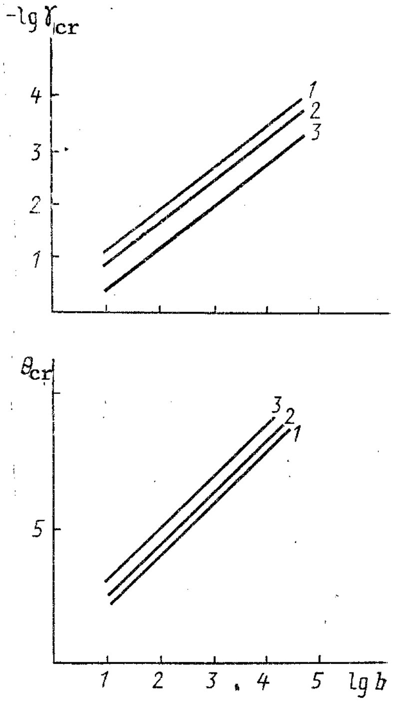


Fig. 3. Effect of the value of the constant component of the coefficient of dielectric losses b on the value of the parameters $\gamma_{cr}$ and $\Theta_{cr}$ ($\Theta_\infty = 0$, Bi = 1): 1) a plane plate; 2) an infinite cylinder; 3) a sphere.

is small: ab << 1. We assume that in the region of the critical temperature the value of the coefficient of dielectric losses itself is small, i.e., for T < $T_{cr}$ $\varepsilon_2(T) \ll 1$. The justification of the given assumption is considered below. However, if $\varepsilon_2 \ll 1$, then

$$h_E = \lambda_0 \sqrt{\varepsilon_1}/\pi\varepsilon_2 \gg 2r.$$

Equation (4) can be written as

$$q = \frac{2\pi}{\lambda_0 \sqrt{\varepsilon_1}} S_0 \varepsilon_2 (T). \tag{7}$$

By using Eq. (7), we rewrite Eq. (1) in a dimensionless form, introducing dimensionless variables: $\theta = \beta(T - T_0)$ and $\eta = x/r$. Equation (1) assumes the form

$$\frac{d^2\Theta}{d\eta^2} = -\gamma (b + \exp \Theta) \tag{8}$$

with the boundary conditions

$$\left.\frac{\partial\Theta}{d\eta}\right|_{\eta=\pm 1} = \mp \mathrm{Bi}\, (\Theta - \Theta_\infty). \tag{9}$$

Here $\gamma = (2\pi S_0 a \beta r^2)/(\lambda_0 k \sqrt{\varepsilon_1})$ is a dimensionless parameter of the volumetric heat evolution; Bi = $\alpha r/k$ is the Biot number.

If b = 0, then the general integral of Eq. (8) is

$$\Theta = \ln \frac{c}{\mathrm{ch}^2 \left( \sqrt{\frac{c\gamma}{2}}\, \eta \right)}. \tag{10}$$

Here c is an arbitrary constant determined from the boundary conditions. For boundary conditions (9) we obtain for c a transcendental equation

$$\sqrt{2c\gamma}\ \mathrm{th}\sqrt{\frac{c\gamma}{2}} = \mathrm{Bi}\left(\ln\frac{c}{\mathrm{ch}^2\sqrt{\frac{c\gamma}{2}}} - \Theta_\infty\right). \tag{11}$$

For the values of $\gamma$ for which (11) has a solution, a steady-state temperature distribution is possible, the form of which is determined by substituting this solution in Eq. (10).

For further analysis it is convenient to introduce instead of the constant of integration c a new quantity $\mu$, related to it by the relationship $c = 2\mu^2/\gamma$. Then Eq. (11) can be written in the form

$$\gamma = 2\exp(-\Theta_\infty)\left(\frac{\mu}{\mathrm{ch}\,\mu}\right)^2 \exp\left(-\frac{2\mu}{\mathrm{i}}\ \mathrm{th}\,\mu\right). \tag{12}$$

The dependence of $\gamma$ on the value of $\mu$ for fixed values of $\theta_\infty$ and Bi is shown in Fig. 1. The fact that this function has an extremum $\gamma_{cr}$ confirms the existence of the values of $\gamma > \gamma_{cr}$ for which there are no solutions of Eq. (11). Therefore, $\gamma_{cr}$ determines a limiting possible steady-state temperature distribution. The corresponding value of $\mu_{cr}$ determines the maximum temperature of the limiting steady state $\Theta_{cr} = \ln(2\mu_{cr}^2/\gamma_{cr})$.

With the use of Eq. (12), we can analyze the effect of the boundary conditions on the value of the critical temperature. Thus, when Bi increases, the heat flow from the surface of the sample increases, and, therefore, the transition to the regime with peaking is realized at larger values of $\Theta_{cr}$ and $\gamma_{cr}$. In the limit for $\mathrm{Bi}\to\infty$ we have $\Theta_{cr} = 1.18$ and $\gamma_{cr} = 0.87$ (Fig. 2).

If $b \neq 0$, then the general integral of Eq. (8) does not have an analytical expression. In agreement with (3) it can be written as

$$\gamma = \frac{1}{2}\left[\int_{\Theta_s}^{\Theta_m}\frac{d\Theta}{\sqrt{b(\Theta_m-\Theta)+\exp\Theta_m-\exp\Theta}}\right]^2.$$

Here $\theta_s$ is determined from the solution of the transcendental equation

$$b(\Theta_m-\Theta_s)+\exp\Theta_m-\exp\Theta_s = \frac{\mathrm{Bi}^2}{2\gamma}(\Theta_\infty-\Theta_s)^2.$$

By using standard methods of numerical integration we can obtain the dependence of $\gamma$ on $\Theta_m$ for the given values of Bi, $\Theta_\infty$, and b. Similarly as for $b = 0$, the maximum of this function $\gamma_{cr}$ determines the limiting possible steady state. The associated value of $\Theta_{cr}$ is the maximum warming-up in the medium before transition to the nonsteady-state regime.

By varying the value of b over a wide range, we can determine the effect of dielectric properties of the medium on the critical temperature (Fig. 3). Each point on the curve $\gamma_{cr} = f(b)$ determines the limiting possible steady-state temperature distribution for the given boundary conditions. Accordingly, the curve itself (Fig. 3) divides the space of parameters ($\gamma$; b) into the region of the steady-state solution of the heat-conduction equation and the region of transition to the thermal regime with peaking.

We now find the conditions for the existence of the steady-state temperature distribution for cylindrical and spherical samples. For these cases the steady-state heat-conduction equation with an exponential function of the source of type (7), written in dimensionless form, has the form [3]:

for an infinite cylinder

$$\frac{\partial^2\Theta}{d\eta^2}+\frac{1}{\eta}\frac{d\Theta}{d\eta} = -\gamma(b+\exp\Theta), \tag{15}$$

for a sphere

$$\frac{\partial^2\Theta}{d\eta^2}+\frac{2}{\eta}\frac{d\Theta}{d\eta}=-\gamma(b+\exp\Theta).$$

Here $\eta = x/r$, where r is the radius of the sample. The boundary conditions agree with (9).

When integrating Eqs. (15) and (16) numerically it is convenient to use auxiliary variables: $\varphi=\eta\sqrt{\gamma\exp\Theta}$ and $\nu=\Theta_m-\Theta$.

Then the differential equations assume the form

$$\frac{d^2\vartheta}{d\varphi^2}+\frac{1}{\varphi}\frac{d\vartheta}{d\psi}-\exp(-\vartheta)=b\exp(-\Theta_m) \tag{17}$$

and

$$\frac{d^2\vartheta}{d\varphi^2}+\frac{2}{\varphi}\frac{d\vartheta}{d\varphi}-\exp(-\vartheta)=b\exp(-\Theta_m) \tag{18}$$

for the cylindrical and spherical samples, respectively. The boundary conditions are

$$\left.\frac{d\vartheta}{d\varphi}\right|_{x=\pm1}=\pm\frac{\mathrm{Bi}(\Theta_m-\Theta_\infty-\vartheta)}{\sqrt{\gamma\exp\Theta_m}}. \tag{19}$$

The solution of Eq. (17) or (18) with the boundary conditions (19) allows us to obtain the dependence of $\gamma$ on $\Theta_m$ for the given Bi, $\Theta_\infty$, and b. The maximum of this function $\gamma_{cr}$, just as for the plane plate, determines the limiting steady-state temperature distribution for a cylinder or sphere. The corresponding value of $\Theta_{cr}$ is the maximum temperature of this state. In Fig. 3, the dependences of $\gamma_{cr}$ and $\Theta_{cr}$ on the value of b are shown for bodies of different geometry.

From the results of calculations it follows that for all $b \geq 1$ the value of the exponential term in (6) for $T \leq T_{cr}$ does not exceed the order of b. Therefore, in the region of critical temperature $\varepsilon_2(T)\approx 2ab\ll 1$, i.e., the assumption taken earlier holds for $T \leq T_{cr}$.

Based on the results obtained, for calculating $\gamma_{cr}$ (in a dimensional form, the threshold value of the power of the SHF radiation) and the value of the critical temperature $\Theta_{cr}$, we can propose the following empirical dependences ($\mathrm{Bi} \leq 5$):

$$\lg\gamma_{cr}=-(0{,}25+0{,}8\lg b-0{,}25n^{3/2})\times$$
$$\times(1-0{,}18\lg\mathrm{Bi}),$$
$$\Theta_{cr}=(1{,}96\lg b+0{,}3n^{3/2})(1+0{,}04\lg\mathrm{Bi})+\Theta_\infty$$

or in a dimensional form

$$S_{cr}=\frac{\lambda_0 k\sqrt{\varepsilon_1}}{2\pi a\beta r^2}\exp[-(0{,}25+0{,}8\lg b-0{,}25n^{3/2})(1-0{,}18\lg\mathrm{Bi})],$$
$$T_{cr}=\frac{1}{\beta}(1{,}96\lg b+0{,}3n^{3/2})(1+0{,}04\lg\mathrm{Bi})+T_\infty.$$

Here n = 0, 1, and 2 for a plane plate, a cylinder, and a sphere, respectively.

Conclusion. It is shown that the nonlinear heat evolution in a dielectric due to the absorption of electromagnetic energy can result at a certain temperature in the development of the nonsteady-state heat regime in the sample.

For dielectric materials, the temperature dependence of the loss factor of which can be approximated by an exponential function, a technique for calculating the critical temperature and the threshold value of the SHF power is developed. The connection between the critical temperature and the thermophysical properties of the dielectric and the conditions of heat exchange with the ambient medium are determined.

Based on the results obtained the empirical dependences are proposed to estimate the value of the critical temperature and the threshold value of the SHF power for the samples of different geometry and different conditions of heat exchange.

The results of calculations can be used in the development of the components of different SHF devices.

## NOTATION

T, temperature; x, coordinate; k, coefficient of heat conduction, r, semi-width of the plate; $\alpha$, coefficient of heat transfer; $T_\infty$, ambient temperature; $T_m$, $T_s$, temperatures in the center and on the surface of the sample; $\Theta_\infty$, dimensionless ambient temperature; $\Theta_m$ and $\Theta_s$, dimensionless temperatures in the center and on the surface of the sample; $S_0$, Poynting vector of the incident wave; $\varepsilon_1$, $\varepsilon_2$, real and imaginary components of the relative dielectric permeability; $\lambda_0$, wavelength of electromagnetic wave in vacuum.

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